





UNIFIED INTERNATIONAL MATHEMATICS OLYMPIAD

CLASS - 10

Question Paper Code : UM9264

KEY

1	2	3	4	5	6	7	8	9	10
С	В	С	А	D	А	С	D	В	А
11	12	13	14	15	16	17	18	19	20
С	D	С	С	А	D	С	С	С	В
21	22	23	24	25	26	27	28	29	30
А	D	В	А	А	А	В	А	С	D
31	32	33	34	35	36	37	38	39	40
A,D	B,C	A,C,D	A,B,D	A,C	С	D	D	D	D
41	42	43	44	45	46	47	48	49	50
В	D	С	D	С	С	D	D	В	А

EXPLANATIONS

...

 \Rightarrow

....

....

MATHEMATICS - 1

01. (C) Given $b^2 = 4ac$

$$\therefore = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{b^2 - b^2}}{2a} = \frac{-b}{2a}$$

- 02. (B) Given $\angle A$, $\angle B$, $\angle C$ are in AP
 - $\therefore \qquad \angle B = \frac{\angle A + \angle C}{2} \implies \angle A + \angle C = 2\angle B$ But $\angle A + \angle B + \angle C = 180^{\circ}$
 - $\therefore 2\angle B + \angle B = 180^{\circ}$

 $3 \angle B = 180^\circ \Longrightarrow \angle B = \frac{180^\circ}{3} = 60^\circ$

03. (C)
$$\triangle AMP \sim \triangle ABC$$
 [\because A. A. similarity]
 $\therefore \frac{AM}{AB} = \frac{MP}{BC}$
 $\Rightarrow \frac{AM}{3AM} = \frac{MP}{12 \text{ cm}}$
 $\therefore MP = 4 \text{ cm}$
Similarly we can prove
 $\triangle ANQ \sim \triangle ABC$
 $\therefore \frac{AM}{AB} = \frac{NQ}{BC}$

$$\frac{2AM}{3AM} = \frac{NQ}{12 \text{ cm}}$$

$$\therefore NQ = \frac{2 \times 12 \text{ cm}}{3} = 8 \text{ cm}$$

$$\therefore MP + NQ = 4 \text{ cm} + 8 \text{ cm} = 12 \text{ cm}$$
O4. (A) Given tan $6\theta = 20^{\circ} - 2\theta$

$$\therefore tan $6\theta = 90^{\circ} - 2\theta$

$$\therefore 6\theta = 90^{\circ} - 2\theta$$

$$\therefore 6\theta = 90^{\circ} - 2\theta$$

$$\therefore 8\theta = 90^{\circ}$$

$$\therefore 8\theta = 80^{\circ}$$

$$\therefore 8\theta = 80^{\circ$$$$

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$$=\frac{4\pm\sqrt{20}}{2}$$

$$=\frac{4\pm2\sqrt{5}}{2}$$

$$=(2\pm\sqrt{5})$$
08. (D) Given $4x^2 + 0x - 1 = 0$
 \therefore $a = 4$ $b = 0$ & $c = -1$
 \therefore $a + b = -\frac{b}{a} = -\frac{0}{4} = 0$
09. (B) Let $\frac{1}{\sqrt{x}} - a \otimes \frac{1}{\sqrt{y}} = b$
 \therefore $2a + 3b = \frac{13}{6}$
 \Rightarrow $12a + 18b = 13 \longrightarrow (1)$
 $4a - 9b = \frac{-19}{6}$
 \Rightarrow $24a - 54b = -19 \longrightarrow (2)$
eq. (1) $\times 2 \Rightarrow 246 + 36b = 26$
 $24a - 54b = -19 \longrightarrow (2)$
eq. (1) $\times 2 \Rightarrow 246 + 36b = 26$
 $24a - 54b = -19 \longrightarrow (2)$
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 $eq. (1) \times 2 \Rightarrow 246 + 36b = 26$
 $24a - 54b = -19 \longrightarrow (2)$
 $(-) (+) (+)$
 $90b = 45$
 $b = \frac{45^{-1}}{90^{-2}} = \frac{1}{2}$
 $12a + 18(\frac{1}{2}) = 13 \longrightarrow (1)$
 $12a + 9 = 13$
 $12a = 13 - 9 = 4$
 $a = \frac{4^{-1}}{32}$
 \therefore $a = \frac{1}{3} = \frac{1}{\sqrt{x}} \Rightarrow \sqrt{x} = 3$
 \therefore $x = 9$

$$\therefore \quad b = \frac{1}{2} = \frac{1}{\sqrt{y}} \Rightarrow \sqrt{y} = 2$$

$$\therefore \quad y = 4$$

$$2x - 5y = 2 \times 9 - 5 \times 4 = 18 - 20 = -2$$

10. (A) Let the three consecutive integers be x,

$$(x + 1) \& (x + 2)$$

Given $x^2 + (x + 1) (x + 2) = 277$

$$\Rightarrow \quad x^2 + x^2 + 3x + 2 - 277 = 0$$

$$\Rightarrow \quad 2x^2 + 3x - 275 = 0$$

$$\Rightarrow \quad 2x^2 + 25x - 22x - 275 = 0$$

$$\Rightarrow \quad x(2x + 25) -11 (2x + 25) = 0$$

$$(2x + 25) (x - 11) = 0$$

$$x - 11 = 0 (OR) \qquad 2x + 25 = 0$$

$$x = 11 \qquad (OR) \qquad 2x = -25$$

$$x = \frac{-25}{2}$$

$$\therefore \quad x = 11 \qquad [x = \frac{-25}{2} \text{ is rejected because}$$

it is not a positive integer]

$$\therefore \quad x + x + 1 + x + 2 = 11 + 12 + 13 = 36$$

11. (C) Given 1001, 1005, _____ 9997 are the
required numbers which are in Arithmetic
progression.

$$\therefore \quad a = 1001, d = 4 \& a_n = 9997$$

$$\therefore \quad a_n = a + (n - 1)d = 9997$$

$$1001 + (n - 1)4 = 9997$$

$$1001 + (n - 1)4 = 9997$$

$$(n - 1) 4 = 9997 - 1001$$

$$n - 1 = \frac{8996}{4} = 2249$$

$$n = 2249 + 1 = 2250$$

$$s_n = \frac{n}{2}[a + a_n]$$

$$= \frac{2250}{2} [1001 + 9997]$$

$$= \frac{2250}{2} \times 10998 \frac{5499}{2}$$

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= 1,23,72,750

12. (D) Given
$$a_{7} = 31$$
 & $a_{1} = -5$
 \therefore $a + 6d = 31$
 $-5 + 6d = 31$
 $6d = 36$
 $d = 6$
 $x_{1} = a + d = -5 + 6 = 1,$
 $x_{2} = x_{1} + d$
 $x_{2} = 1 + 6 = 7$
 $x_{3} = x_{2} + d$
 $x_{3} = 7 + 6 = 13$
 \therefore $x_{4} = x_{3} + d = 13 + 6 = 19$
 $x_{5} = x_{4} + d = 19 + 6 = 25$
 \therefore $x_{1} + x_{2} + x_{3} + x_{4} + x_{5} = 1 + 7 + 13 + 19 + 25$
 $= 65$
13. (C) A remainder is always less than devisor
but it can be zero also
 \therefore $a = bq + r$
where $0 \le r < b$
 \therefore Option (C) is correct
14. (C) Given (-1) is the zero of $p(x) = x^{3} + ax^{2} + bx + c$
 $p(-1) = (-1)^{3} + a(-1)^{2} + b(-1) + c = 0$
 $-1 + a - b + c = 0$
 $c = (b - a + 1)$
But $\alpha\beta\gamma = \frac{-c}{a}$
 \Rightarrow $-1 \times \beta r = \frac{-(b - a + 1)}{1}$
 \therefore $\beta r = (b - a + 1)$
15. (A) Let the present ages of the father and the
son be 'x' years and 'y' years respectively
Five years ago, their ages were $(x - 5)$
years and $(y - 5)$ years
According to the problem
 $(x - 5) = 7(y - 5)$
 \Rightarrow $x - 5 + 35 - 7y = 0$
 \Rightarrow $x - 7y + 30 = 0$

 \Rightarrow x = 7y - 30..... (1) Five years later, their ages will be (x + 5)years and (y + 5) years According to the problem (x + 5) = 3(y + 5) \Rightarrow x + 5 = 3y + 15 \Rightarrow x = 3y + 15 - 5 x = 3y + 10..... (2) \Rightarrow From (1) and (2), we have 7y - 30 = 3y + 104y = 40 \Rightarrow \Rightarrow *y* = 10 $\Rightarrow x = 7y - 30$ = 7(10) - 30= 40 The present ages of the father and the ... son are 40 years and 10 years respectively 16. (D) Given 63, 65, 67... are in AP. a = 63, d = 65 – 63 = 2 $a_n = a+(n-1)d = 63 + (n-1)(2) = 63 + 2n$ $-2 = 61 + 2n \longrightarrow (1)$ Given 3, 10, 17... are in AP b = 3 D = 10 - 3 = 7 $b_n = b + (n-1) D = 3 + (n-1) 7 = 3 + 7n - 7$ = 7n – 4 Given a = b = 61 + 2n = 7n - 4 \Rightarrow 61 + 4 = 7n - 2n5n = 65 n = 13 17. (C) Given ABCD is a rectangle \therefore BD = AC = $\sqrt{(11 + 10)^2 + (15 + 5)^2}$ $=\sqrt{21^2+20^2}$ $=\sqrt{441+400}$ $=\sqrt{841}$ = 29

18. (C) In quadrilaterals ABCD and PQRS

$$\frac{7}{20} = \frac{z}{16\frac{2}{3}} \Rightarrow \frac{7}{20} = \frac{3z}{50}$$

$$\Rightarrow z = \frac{35}{6} = 5\frac{5}{6} \text{ units}$$
19. (C) Given area of $\triangle ABC = 70 \text{ square units}$

$$\therefore \frac{1}{2} |\lambda(2\lambda - 6+2\lambda) - \lambda + 1(6-2\lambda - 2-2\lambda) - 4 - \lambda(2-2\lambda - 2\lambda)| = 70$$

$$|\lambda(4\lambda - 6) - \lambda + 1(4) - 4 - \lambda(2-4\lambda)| = 2 \times 70$$

$$\Rightarrow 4\lambda^2 - 6\lambda - 4\lambda + 4 - 8 + 16\lambda - 2\lambda + 4\lambda^2 = \pm 140$$

$$8\lambda^2 + 4\lambda - 4 = \pm 140$$

$$4(2\lambda^2 + \lambda - 1) = \pm 140$$

$$2\lambda^2 + \lambda - 1 = \pm \frac{140}{4} = 35$$

$$2\lambda^2 + \lambda - 1 = 35$$

$$2\lambda^2 + \lambda - 36 = 0$$

$$2\lambda^2 + 9\lambda - 8\lambda - 36 = 0$$

$$\lambda(2\lambda + 9) - 4(2\lambda + 9) = 0$$

$$(2\lambda + 9) (\lambda - 4) = 0$$

$$\lambda = 4 \in z \text{ but } \lambda = \frac{-9}{2} \notin Z$$
(or)

$$2\lambda^2 + \lambda - 1 = -35$$

$$2\lambda^2 + \lambda + 34 = 0$$

$$\Delta = -b^2 - 4ac$$

$$= (1)^2 - 4 \times 2 \times 34$$

$$= 1 - 272$$

$$\Delta = -271$$

$$\Delta < 0 \Rightarrow \text{ No real roots}$$

$$\therefore \text{ One integer satisfies } \lambda \text{ value}$$
20

l 20. (B) $\angle BAP = \angle PAO - 35^{\circ} = 90^{\circ} - 35^{\circ} = 55^{\circ}$ [∵ A tangent is perpendicular to the radius] In $\triangle APB$, $AP = PB \implies \angle ABP = \angle BAP = 55^{\circ}$ In \triangle APB, 55° + 55° + \angle APB = 180° ∠APB = 180° - 110° = 70° (A) Given tan (A – B) = $\frac{1}{\sqrt{3}}$ = tan 30° \therefore A – B = 30° \longrightarrow (1) Given $\cos(A + B) = 0 = \cos 90^{\circ}$ $A + B = 90^{\circ} \longrightarrow (2)$ eq. $(1) + (2) \Rightarrow \angle A - \angle B' + \angle A + \angle B$ $= 30^{\circ} + 90^{\circ}$ 2∠A = 120° ∠A = 60° $60^\circ + \angle B = 90^\circ \longrightarrow (2)$ ∠B = 90° - 60° = 30° $\angle A + 2 \angle B = 60^\circ + 2 \times 30^\circ$... $= 60^{\circ} + 60^{\circ} = 120^{\circ}$ Since, quadrilateral circumscribing a 2. (D) circle then opposite sides subtends supplementary angles at the centre of the circle. $\angle AOB + \angle COD = 180^{\circ}$ *.*.. 125° + ∠COD = 180° ∠COD = 180° – 125° = 55° B. (B) Volume of cuboid = Volume of cylinder lbh = π r²h \Rightarrow $r^2 = \frac{44 \times 30 \times 15 \times 7}{22 \times 28}$ \Rightarrow r = 15 cm Hence radius of the cylinder is equal to 15 cm

24. (A) Diameter of big semicircle = (42 m + 7 m + 7 m) ÷ 56 m Radius of big semicircle = $\frac{56 \text{ m}}{2}$ = 28 m Length of rectangle = 126 m - 28 m - 28 m = 70 m Total area = $(\frac{22}{7} \times 28 \text{ m} \times 28 \text{ m}) + (70 \text{ m})$ × 56 m) = 2464 m² + 3920 m² $= 6384 \text{ m}^2$ Radius of small semicircle $= 42 \text{ m} \div 2 = 21 \text{ m}$ Unshaded area = $(\frac{22}{7} \times 21 \text{ m} \times 21 \text{ m}) +$ (70 m × 42 m) = 1386 m² + 2940 m² $= 4326 \text{ m}^2$ Area of the running track = 6384 m^2 – ... 4326 m² = 2058 m² 25. (A) Let AB be the height of the building (x)m), BC be the distance of the observer from the foot of the building and the angle of elevation is 30°. x m Then tan 30° = $\frac{x}{100}$ $x = 100 \times \tan 30^\circ = \frac{100}{\sqrt{3}} \text{m} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{100 \times 1.73}{3}$ \Rightarrow = 57.66 m

26. (A) Given
$$\sec\theta + \tan\theta = 2$$
(1)
but $\sec^2\theta - \tan^2\theta = 1$
 $(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$
 $2(\sec\theta - \tan\theta) = 1$
 $\sec\theta - \tan\theta = \frac{1}{2}$ (2)
 $\sec\theta + \tan\theta = 2$ (1)
 $\sec\theta - \tan\theta = \frac{1}{2}$ (2)
 $2\tan\theta = 2 - \frac{1}{2} = \frac{4-1}{2} = \frac{3}{2}$
 $\therefore \tan\theta = \frac{3}{2} \times \frac{1}{2} = \frac{3}{4}$
27. (B) Given $\sqrt{7}$, $3\sqrt{7}$, $5\sqrt{7}$ are in AF
 \therefore $a = \sqrt{7} d = 3\sqrt{7} - \sqrt{7} = 2\sqrt{7}$
 $s_n = \frac{n}{2} [2a + (n - 1)d]$
 $= \frac{n}{2} [2\sqrt{7} + (n - 1)2\sqrt{7}]$
 $= \frac{n}{2} [2\sqrt{7} + 2\sqrt{7} n - 2\sqrt{7}]$
 $= \frac{n}{2} \times n \times 2\sqrt{7}$
 $s_n = n^2\sqrt{7}$

28. (A)
$$a = 5, b = -2\sqrt{6} c = -2$$

 $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
 $= \frac{-(-2\sqrt{6} \pm \sqrt{(-2\sqrt{6})^2 - 4 \times 5 \times -2}}{2(5)}$
 $= \frac{2\sqrt{6} \pm \sqrt{24 + 40}}{10}$
 $= \frac{2\sqrt{6} \pm 8}{10} = \frac{2(\sqrt{6} \pm 4)}{10}$
 $= \frac{4 + \sqrt{6}}{5} (OR) \frac{-4 + \sqrt{6}}{5}$
29. (C) 144) 180 (1
 $\frac{144}{36}$) 144 (4
 $\frac{144}{0}$
 \therefore HCF of 144 and 180 = 36
Given 7m + 113 = 36
7m = 36 - 113
7m = -77
 $m = \frac{-77^{11}}{7_1}$
 $m = -11$
30. (D) Required polynomial = $k[x^2 - x(\alpha + \beta) + \alpha\beta]$ where k is any real number other
than zero.
 $= k[x^2 - x(-3) - 10]$
 $= k(x^2 + 3x - 10)$
 $= x^2 + 3x - 10 (OR) (\frac{x^2}{2} + \frac{3x}{2} - 5)$

MATHEMATICS - 2

31. (A, D)

Given P divides the Join of AB in the ratio 1:2A(-3, 2) B(9, 5) 1:2

$$\mathsf{P} = \left(\frac{\mathsf{m}_1 x_2 + \mathsf{m}_2 x_1}{\mathsf{m}_1 + \mathsf{m}_2}, \frac{\mathsf{m}_1 y_2 + \mathsf{m}_2 y_1}{\mathsf{m}_1 + \mathsf{m}_2}\right) = \left(\frac{9-6}{1+2}, \frac{5+4}{3}\right)$$

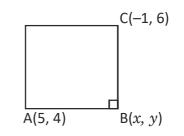
= (1, 3)

Given Q divides the join of AB in the ratio 2:1

$$\therefore Q = \left(\frac{9 \times 2 - 3 \times 1}{3}, \frac{2 \times 5 + 2 \times 1}{3}\right)$$

32. (B, C)

Given A(5, 4) & (-1, 6)



let B be (x, y)

AB = BC [Given ABCD is a square]

$$\sqrt{(x-5)^2+(y-4)^2} = \sqrt{(x+1)^2+(y-6)^2}$$

squaring on both sides.

$$x^{z'} - 10x + 25 + y^{z'} - 8y + 16 = x^{z'} + 2x + 1 + y^{z'} - 12y + 36$$

$$-10x - 2x - 8y + 12y = 37 - 25 - 16$$

$$-12x + 4y = -4$$

$$3x - y = 1$$

$$3x - 1 = y$$

But AC² = AB² + BC²

$$(5 + 1)^{2} + (4 - 6)^{2} = (x - 5)^{2} + (y - 4)^{2} + (x + 1)^{2} + (y - 6)^{2}$$

$$36 + 4 = x^{2} - 10x + 25 + y^{2} - 8y + 16 + x^{2} + 2x + 1 + y^{2} - 12y + 36$$

$$2x^{2} + 2y^{2} - 8x - 20y + 78 = 40$$

$$2(x^{2} + y^{2} - 4x - 10y + 39) = 40$$

$$x^{2} + y^{2} - 4x - 10y + 39 = \frac{40^{20}}{2}$$

$$x^{2} + (3x - 1)^{2} - 4x - 10 (3x - 1) + 39 = 20$$

$$\therefore x^{2} + 9x^{2} - 6x + 1 - 4x - 30x + 10 + 39 = 20$$

$$10x^{2} - 40x + 30 = 0$$

$$10(x^{2} - 4x + 3) = 0$$

$$x^{2} - 4x + 3 = 0$$

$$x^{2} - 3x - x + 3 = 0$$

$$x(x - 3) - 1(x - 3) = 0$$

$$(x - 3) (x - 1) = 0$$

$$x = 3 (or) x = 1$$
If $x = 1$ then $y = 3x - 1 = 2$
one vertex = (1, 2)
If $x = 3$ then $y = 3x - 1 = 8$
other vertex = (3, 8)
33. (A, C, D)
Given lines are parallel $\Rightarrow \frac{a_{1}}{a_{2}} = \frac{b_{1}}{b_{2}} \neq \frac{c_{1}}{c_{2}}$

$$\frac{3^{2}}{b_{2}^{2}} = \frac{+1}{+2} \neq \frac{+p}{+5}$$

$$\therefore p \neq \frac{5}{2}$$

$$\therefore ' p' \text{ can be real number except } \frac{5}{2}$$

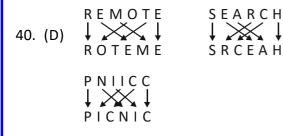
$$\therefore p = 5 \text{ or } - 5 (\text{ or }) 0$$
34. (A, B, D)
Option A sum of the roots $= \frac{-b}{a} = \frac{-3}{-1} = 3$
Option B sum of the roots $= \frac{-b}{a} = \frac{-15}{5} = -3$
Option D sum of the roots $= \frac{-b}{a} = \frac{-(-9)}{3} = \frac{9}{3} = 3$

35. (A, C) Three units from B is C (5, 0)let A be (x, y)Given AB = AC $AB^2 = AC^2$ $(x-2)^2 + (y-0)^2 = (x-5)^2 + (y-0)^2$ $x^{z'} - 4x + 4 + y^{z'} = x^{z'} - 10x + 25 + y^{z'}$ 10x - 4x = 25 - 4 = 216x = 21 $x = \frac{21^7}{62} = \frac{7}{2}$ $\left(\frac{7}{2}, \mathcal{Y}\right)$ is 3 units from (2, 0) $\therefore \sqrt{\left(2-\frac{7}{2}\right)^2+\left(0-y\right)^2} = 3$ $\sqrt{\left(\frac{4-7}{2}\right)^2 + y^2} = 3$ Squaring on both sides $\left(\frac{-3}{2}\right)^2 + y^2 = 9$ $y^2 = 9 - \frac{9}{4} = \frac{36 - 9}{4}$ $y = \pm \sqrt{\frac{27}{4}} = \pm \frac{3\sqrt{3}}{2}$

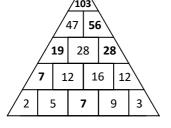
REASONING

- 36. (C) The shapes are moving around the points of the polygon. The circle and arrow are both moving anti-clockwise 2 spaces, and the square is moving 3 spaces in a clockwise direction
- 37. (D) Except option (D), remaining options are equally portioned.
- 38. (D) when P is selected that Z should also be selected and when R is selected than T should also be selected. Thus Z and T will be the other two members of the group. The only option that does not have Z and T is the option (D). So the correct answer is (D).
- 39. (D) More than 17

8 big square + 8 small square + 2 square in middle = 18



41. (B)



- 7155 × 7156 6217 × 2217 (D) .24
- 43. (C) A = East, B = West, D = East, C = West

